Update..Students are not responsible for the last five pages with questions on limits

Welcome to AP Calculus AB class!

Attached is a summer homework packet, which will be due the first day of Calculus class in August. The material in the packet should topics you learned in Algebra II and Precalculus.

You will turn in the packet the first day of Calculus class, and it will count as 60 points of your first test grade. It will be graded for completion. You are not responsible for the Preview of Calculus Section problems, even though I strongly encourage you to try yourself. We will thoroughly explain and study limits and derivate during the first two week of class, so you will have time to fill in those answers. During the first week of class, you will take a test on the material in the packet that will count for 40 points of your first test grade.

Have a great summer and enjoy the beauty of calculus!

Summer Work for AP Calculus AB White Station High School

I. Exponential and Logarithmic Functions

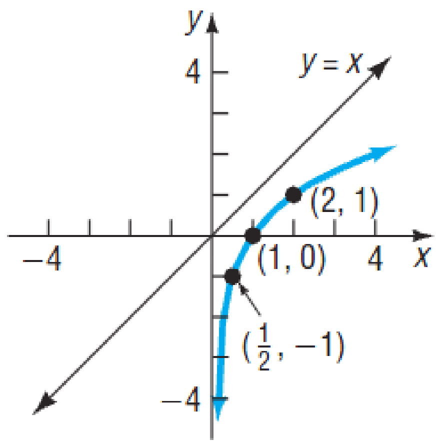
1. *For the given functions f and g, find*

(a) $\left(f \circ g\right)(2)$ (b) $\left(g \circ f\right)(-2)$ (c) $\left(f \circ f\right)(4)$ (d) $\left(g \circ g\right)(-1)$

$f(x) = 1 - 3x^2$; $g(x) = \sqrt{4-x}$

2. *Find* $f \circ g, g \circ f, f \circ f,$ *and* $g \circ g$ *for each pair of functions. State the domain of each composite function.* $f(x) = \sqrt{x-3}$; $g(x) = \dfrac{3}{x}$

3. State why the given graph of a function is one-to-one. Then draw the graph of the inverse function f^{-1}. For convenience (and as a hint), the graph of $y = x$ is also given.



4. *The function f is one-to-one. Find the inverse of each function and check your answer.*

a) $f(x) = \dfrac{2-x}{3+x}$

b). $f(x) = \dfrac{3}{x^{1/3}}$

5. Given that $f(x) = 3^x$ and $g(x) = \log_3 x$, evaluate

(a) $f(1)$ (b) $g(81)$ (c) $f(-4)$ (d) $g\left(\frac{1}{243}\right)$

6. *Write each expression as the sum and/or difference of logarithms. Express powers as factors.*

a) $\log_5 \left(u\sqrt{v}\right)^3, u > 0, v > 0$

b) $\ln\left(\frac{x\sqrt{x^2+1}}{x-3}\right), x > 0$

7. *In Problems a-d, write each expression as a single logarithm.*

a) $2\log_2 x - 3\log_2 y$

b) $-2\log_3\left(\frac{1}{x}\right)+\frac{1}{3}\log_3\sqrt{x}$

c) $\log\left(x^2-9\right)-\log\left(x^2+7x+12\right)$

d) $2\log 2+3\log x-\frac{1}{2}\left[\log(x+3)+\log(x-2)\right]$

8. *In Problems below, solve each equation. Express irrational solutions in exact form and as a decimal rounded to 3 decimal places. Verify your results using a graphing utility.*

a) $4^{x-x^2}=\frac{1}{2}$

b) $\log_{\sqrt{2}} x=-6$

c) $5^{x+2}=7^{x-2}$

d) $2^{x+1}\cdot 8^{-x}=4$

e) $\log(7x-12)=2\log x$

f) $e^{1-2x}=4$

g) $4^{x}-14\cdot 4^{-x}=5$

9. Suppose that $f(x)=\log_3(x+1)-4$

   a) Graph *f*.

   b) What is $f(8)$? What point is on the graph of *f*?

   c) Solve $f(x)$ = -3. What point is on the graph of *f*?

d) Based on the graph drawn in part (a), solve $f(x) < 0$.

e) Find $f^{-1}(x)$. Graph f^{-1} on the same Cartesian plane as f.

10. A child's grandparents wish to purchase a bond that matures in 18 years to be used for her college education. The bond pays 4% interest compounded semiannually. How much should they pay so that the bond will be worth $85,000 at maturity?

11. Suppose the population of a newly discovered insect grows according to the logistic growth model $P(t) = \dfrac{50000}{1+25e^{-0.04t}}$ where P represents the population and t represents the time in years.

a) How many insects were originally discovered?

b) Determine the maximum population of the insect population.

c) Use a graphing utility, graph $P = P(t)$.

d) When will the population reach 20,000 insects?

II. Analytic Trigonometry

1. Find the exact value, if any, of each composite function. If there is no value, say it is "not defined." Do not use a calculator.

   a) $\sin^{-1}\left[\sin\left(-\frac{\pi}{8}\right)\right]$

   b) $\tan\left[\cos^{-1}\left(-\frac{3}{5}\right)\right]$

2. Find the inverse function of each function f. Find the range of f and the domain and range of $f(x) = 2\sin(-x+1),\ 1-\frac{\pi}{2} \le x \le 1+\frac{\pi}{2}$

3. Find the exact value of each expression. $\sin\left(\cos^{-1}\frac{4}{5}+\sin^{-1}\left(-\frac{2}{3}\right)\right)$

4. Solve each equation on the interval $0 \le \theta \le 2\pi$.

a) $2\cos\theta + \sqrt{2} = 0$

b) $\cos(2\theta) = \sin\theta$

c) $\sin(2\theta) - \sin\theta - 2\cos\theta + 1 = 0$

d) $8 - 12\sin^2\theta = 4\cos^2\theta$

e) $1 + \sqrt{3}\cos\theta + \cos(2\theta) = 0$

IV. **Analytic Geometry**

1. *In Problems below, identify each equation. If it is a parabola, give its vertex, focus, and directrix; if it is an ellipse, give its center, vertices, and foci; if it is a hyperbola, give its center, vertices, foci, and asymptotes.*

   a) $\frac{y^2}{25} - x^2 = 1$

   b) $4x^2 + y^2 + 8x - 4y + 4 = 0$

   c) $4x^2 + 9y^2 - 16x + 18y = 11$

2. In Problems below, find an equation of the conic described. Graph the equation by hand.

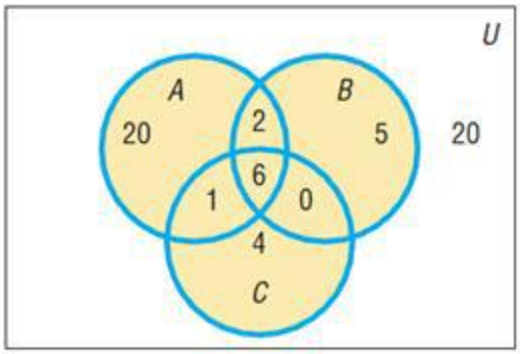
   a) Ellipse; center at (-1, 2); focus at (0, 2); vertex at (2, 2)

b) Parabola; focus at (3, 6); directrix the line y = 8
c) Hyperbola; vertices at (-3, 3) and (5, 3); focus at (7, 3)

3. In Problems below, identify the conic that each polar equation represents and graph it.

V. Counting and Probability

1. In Problems below, use the information supplied in the figure.



a) How many are in *B*?

b) How many are in *A* or *C*?

c) How many are not in *A*?

d) How many are in *A* and *B*?

e) How many are in *C* but not in *B*?

f) How many are in neither *B* nor *C*?

VI. A Preview of Calculus

1. *In Problems below, find the limit.*

a) $\lim_{x \to -2} \left(x^3 + 1\right)^2$

b) $\lim_{x \to -3} (15 - 3x)^{-3/2}$

c) $\lim_{x \to -3} \dfrac{x^2 + 2x - 3}{x^2 - 9}$

d) $\lim_{x \to 2^+} \dfrac{x^2 - 4}{x^3 - 8}$

e) $\lim_{x \to 1} \dfrac{x^3 - 1}{x^3 - x^2 + 3x - 3}$

2. *In Problems below, determine whether f is continuous at c.*

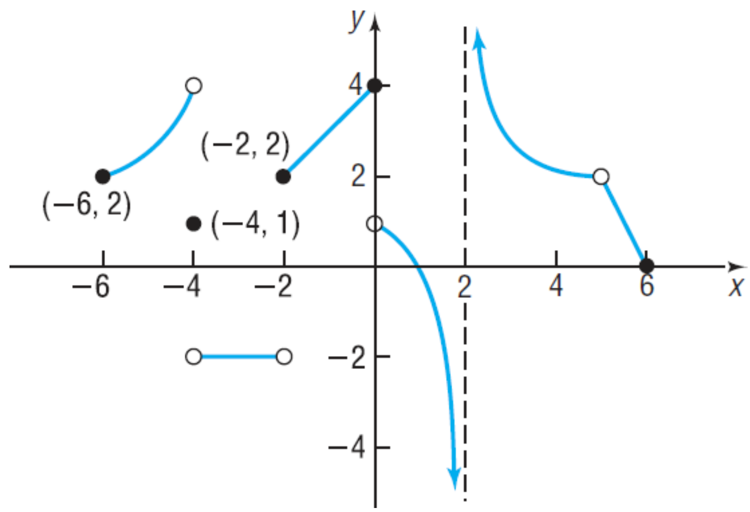
a) $f(x) = \dfrac{x^2 + 6x}{x^2 - 6x}$ $\quad c = 0$

b) $f(x) = \begin{cases} \dfrac{x^2 + 6x}{x^2 - 6x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ $\quad c = 0$

c) $f(x)=\begin{cases}\dfrac{x^2+6x}{x^2-6x} & \text{if } x\neq 0\\ -1 & \text{if } x=0\end{cases}\qquad c=0$

3. *In Problems below, use the accompanying graph of y = f(x).*

a) Find $f(-2)$ and $f(6)$.



b) Find $\lim\limits_{x\to -2^-} f(x)$.

c) Find $\lim\limits_{x\to -2^+} f(x)$.

d) Does $\lim\limits_{x\to 2} f(x)$ exist? If it does, what is it?

e) Does $\lim\limits_{x\to 5} f(x)$ exist? If it does, what is it?

f) Is f continuous at -4?

g) Is f continuous at 2? Is f continuous at 5?

4. *In Problems below, find the slope of the tangent line to the graph of f at the given point. Graph f and the tangent line.*

$$\text{Slope of tangent line} = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

a) $f(x)=3x^2-6x$ at (0,0)

b) $f(x)=2x^2+5x-3$ at (1, 4)

c) $f(x)=x^3-x^2$ at (1, 0)

5. *In Problems below, find the derivative of each function at the number indicated.*

Derivative of the function *f(x)* at *x:* $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

a) $f(x)=-4+3x^2$ at 1

b) $f(x) = 3x^2 - 4x + 1$ at 2

6. **Instantaneous Speed of a Parachutist** The following data represent the distance s (in feet) that a parachutist has fallen over time t (in seconds).

Time, t (in Seconds)	Distance, s (in Feet)
1	16
2	64
3	144
4	256
5	400

(a) Find the average speed from t = 1 to t = 4 seconds.

(b) Find the average speed from t = 1 to t = 4 seconds.

(c) Find the average speed from t = 1 to t = 2 seconds.

(d) Using a graphing utility, find the power function of best fit.

7. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

8 A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals, as shown. What dimensions should be used so that the enclosed area will be a maximum?

9 A rectangle is bounded by the x-axis and the semicircle as shown. What length and width should the rectangle have so that its area is a maximum?